Problem 1.

(a)

To evaluate the definite integral

We can use integration by parts:

Using the composite trapezoidal method, we approximate this answer numerically:

A screen shot of a computer code

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(b) See print out above.

(c) According to the theorem of precision of trapezoid rule, since f(x) has a continuous second derivative on the interval [-1,1], the error

In the program above, each time the step size is made smaller by a factor of 2, the error decreases by a factor of 4, which is consistent with an error of O(h^2).

(d) Programming the Romberg Algorithm, we get the approximations:

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(e) Above we have the ratios between the previous and new Romberg approximation of the integral at each step. Theoretically, as has continuous derivatives of all orders, the theory predicts R(n,1) should have an error of This agrees with our program, in which halving the step size makes the error 16 times smaller.

Code Printout:

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Problem 2.

Suppose for a function , the error of our trapezoidal approximation takes the form:

+… **(I)**

Then note that:

+… **(II)**

Then taking 8 times **(I)** minus **(II)** divided by 8, we get:

Therefore we should define R(n,1) as:

.

Problem 3

Given the nodes:

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 0 | 1 | 10 |
| 1 | 1.25 | 8 |
| 2 | 1.5 | 7 |
| 3 | 1.75 | 6 |
| 4 | 2 | 5 |

The composite Simpson rule give us:

Problem 4(a)A math equations on a notebook

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(b)A notebook with math equations on it

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(c) The Gaussian Quadrature formulas with 2 and 3 interpolating nodes give an approximation of the integral

Accurate to 2 and 4 decimal places respectively.

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Code:

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